Δ to N transition and Δ form factors in Lattice QCD

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THE CYPRUS INSTITUTE

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5. **Δ form factors**
   - Δ electromagnetic form factors
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   - Pseudoscalar Δ couplings
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Motivation

- Δ to Nucleon transition form factors:
  - Electromagnetic form factors are known experimentally; In particular G^*_{M1} is precisely measured $\rightarrow$ can we reproduce it from lattice QCD?
  - Quadrupole $\gamma^* N \rightarrow \Delta$ form factors G_{E2} and G_{C2} may indicate deformation in the nucleon/Δ $\rightarrow$ can we calculate them from lattice QCD?
  - Provide input on axial-vector FFs
    - ★ Understand the q^2-dependence of axial form factors C^A_5 and C^A_6 that correspond to the nucleon axial form factor G_A and nucleon induced pseudoscalar form factor G_p
    - ★ Provide important input for phenomenological models builders and for chiral effective theories
  - Evaluate $\pi N\Delta$ coupling and examine the non-diagonal Goldberger-Treiman relation
- Form factors of the Δ:
  - Difficult to measure experimentally
  - Electromagnetic form factors $\rightarrow$ magnetic moment of Δ, quadrupole moment $\Longrightarrow$ obtain information on its charge distribution in the infinite momentum frame (talk by M. Vanderhaeghen)
  - Study the axial-vector and pseudoscalar form factors $\Longrightarrow$ New features arise e.g. two Goldberger-Treiman relations

$\Longrightarrow$ Calculate within lattice QCD the form factors of the nucleon/Δ system $\rightarrow$ global fit

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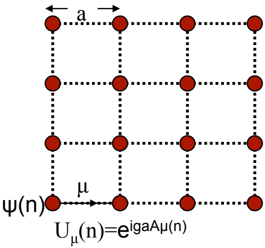
QCD on the lattice



- Discretization of space-time in 4 Euclidean dimensions $\rightarrow$ simplest isotropic hypercubic grid:
  $\Longrightarrow$ Rotation into imaginary time is the most drastic modification
  Lattice acts as a non-perturbative regularization scheme with the lattice spacing a providing an ultraviolet cutoff at $\pi/a \rightarrow$ no infinities
- Gauge fields are links and fermions are anticommuting Grassmann variables defined at each site of the lattice. They belong to the fundamental representation of SU(3)
- Construction of an appropriate action such that when $a \rightarrow 0$ (and Volume$\rightarrow \infty$) it gives the continuum theory
- Construction of the appropriate operators with their renormalization to extract physical quantities
- We take spacing $a = a_S = a_T$ and size $N_S \times N_S \times N_S \times N_T$, $N_T > N_S$ (Hadron Spectrum Collaboration considers anisotropic lattices with $a_T \sim a_S/3 \rightarrow$ better suited for study of excited states)

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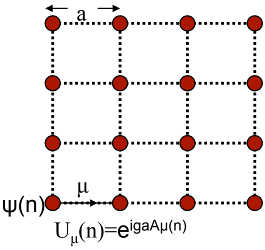

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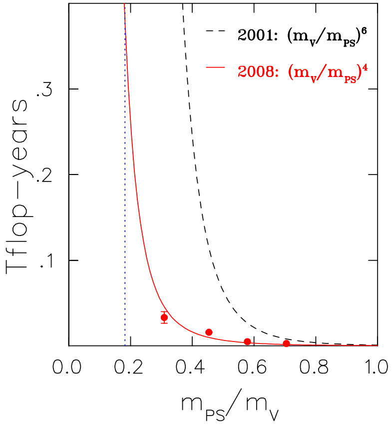
Lattice artifacts

- **Finite Volume:**
  1. Only discrete values of momentum in units of $2\pi/N_S$ are allowed.
  2. Finite volume effects need to be studied $\rightarrow$ Take box sizes such that $L_S m_\pi \gtrsim 3.5$.
- **Finite lattice spacing:** Need at least three values of the lattice spacing in order to extrapolate to the continuum limit.
- q^2**-values:** Fourier transform of lattice results in coordinate space taken numerically $\rightarrow$ for large values of momentum transfer results are too noisy $\Longrightarrow$ Limited to $Q^2 = -q^2 \sim 2$ GeV^2.
  Studies to extend to larger values, H.-W. Lin, *et al.*, arXiv:1005.0799, H.-W. Lin and S. D. Cohen, arXiv:1104.4329

Computational cost

Simulation cost: $C_{\rm sim} \propto \left(\frac{300\text{MeV}}{m_\pi}\right)^{c_m} \left(\frac{L}{2\text{fm}}\right)^{c_L} \left(\frac{0.1\text{fm}}{a}\right)^{c_a}$



L=2.1 fm, a=0.089 fm, K. Jansen and C. Urbach, arXiv:0905.3331

Coefficients c_m, c_L and c_a depend on the discretized action used for the fermions.
State-of-the-art simulations use improved algorithms:

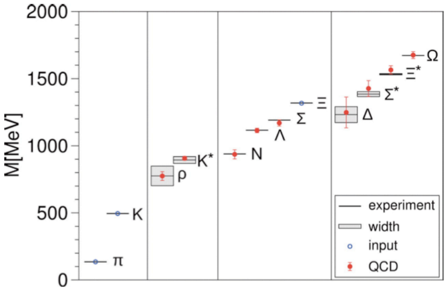
- Mass preconditioner, M. Hasenbusch, Phys. Lett. B519 (2001) 177
- Multiple time scales in the molecular dynamics updates

$\Longrightarrow$ for twisted mass fermions: $c_m \sim 4$, $c_L \sim 5$ and $c_a \sim 6$.

- Simulations at physical quark masses, $a \sim 0.1$ fm and $L \sim 5$ fm require $\mathcal{O}(10)$ Pflop.Years.
- The analysis to produce physics results requires $\mathcal{O}(1)$ Pflop.Year.
- After post-diction of well measured quantities the goal is to predict quantities that are difficult or impossible to measure experimentally.

Mass of low-lying hadrons

$N_F = 2 + 1$ smeared Clover fermions, BMW Collaboration, S. Dürr et al. Science 322 (2008)
$N_F = 2$ twisted mass fermions, ETM Collaboration, C. Alexandrou et al. PRD (2008)



- BMW with $N_F = 2 + 1$:
  - 3 lattice spacings: $a \sim 0.125, 0.085, 0.065$ fm set by m_Ξ
  - Pion masses: $m_\pi \gtrsim 190$ MeV
  - Volumes:$m_\pi^{\min} L \gtrsim 4$
- ETMC with $N_F = 2$:
  - 3 lattice spacings: $a = 0.089, 0.070, a = 0.056$ fm, set my m_N
  - $m_\pi \gtrsim 260$ MeV
  - Volumes:$m_\pi^{\min} L \gtrsim 3.3$

Good agreement between different discretization schemes $\Longrightarrow$ Significant progress in understanding the masses of low-lying mesons and baryons

$\rightarrow$ For Δ to N and Δ form factors we will use domain wall fermions (DWF)

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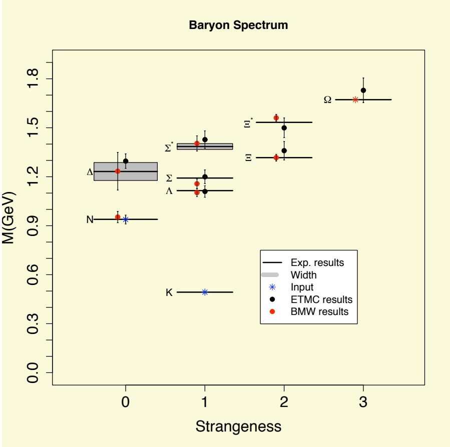

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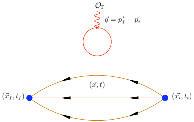
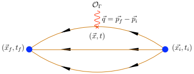
Nucleon form factors

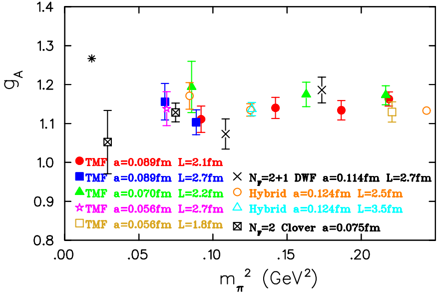
Experimental measurements since the 50's but still open questions $\rightarrow$ high-precision experiments at JLab.

- The electric and magnetic Sachs form factors:
  $G_E(q^2) = F_1(q^2) - \frac{q^2}{(2m)^2} F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$
  - Many lattice studies down to lowest pion mass of $m_\pi \sim 300$ MeV$\Longrightarrow$ Lattice data in general agreement, but still slower q^2-slope
  - Disconnected diagrams neglected so far





- Axial-vector FFs: $A^a_\mu = \bar\psi \gamma_\mu \gamma_5 \frac{\tau^a}{2} \psi(x)$
  $\Longrightarrow \frac{1}{2}\left[\gamma_\mu \gamma_5 G_A(q^2) + \frac{q^\mu \gamma_5}{2m} G_p(q^2)\right]$



C. A. *et al.* (ETMC), Phys. Rev. D83 (2011) 045010

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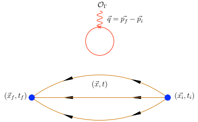

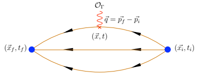

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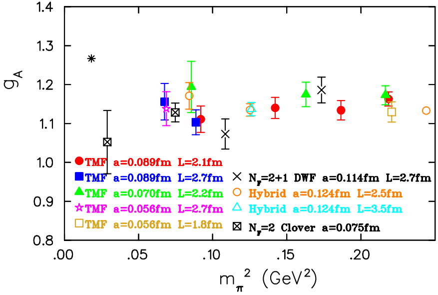
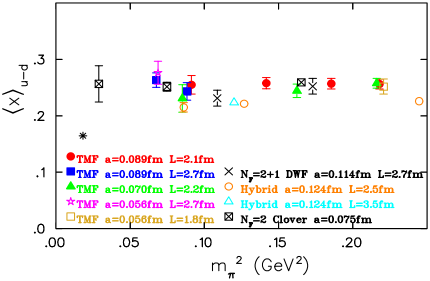
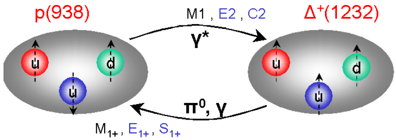


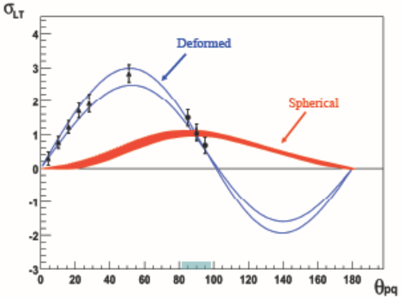


Similar discrepancy also for the momentum fraction, C. A. *et al.* (ETMC), arXvi:1104:1600

$N\gamma^* \to \Delta$ form factors

- A dominant magnetic dipole, M1
- An electric quadrupole, E2 and a Coulomb, C2 signal a deformation in the nucleon/Δ
- 1/2-spin particles have vanishing quadrupole moment in the lab-frame
- Probe nucleon shape by studying transitions to its excited Δ-state
- Difficult to measure/calculate since quadrupole amplitudes are sub-dominant





- $R_{EM}(\text{EMR}) = -\frac{G_{E2}(Q^2)}{G_{M1}(Q^2)}$,

  $R_{SM}(\text{CMR}) = -\frac{|\vec{q}|}{2m_\Delta}\frac{G_{C2}(Q^2)}{G_{M1}(Q^2)}$,

  in lab frame of the Δ.
- Precise data strongly "suggesting" deformation in the Nucleon/Δ
  At $Q^2 = 0.126$ GeV^2:
  EMR=$(-2.00 \pm 0.40_{\rm stat+sys} \pm 0.27_{\rm mod})$%,
  CMR=$(-6.27 \pm 0.32_{\rm stat+sys} \pm 0.10_{\rm mod})$%

C. N. Papanicolas, Eur. Phys. J. A18 (2003); N. Sparveris *et al.*, PRL **94**, 022003 (2005)

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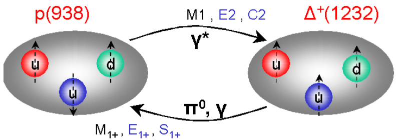
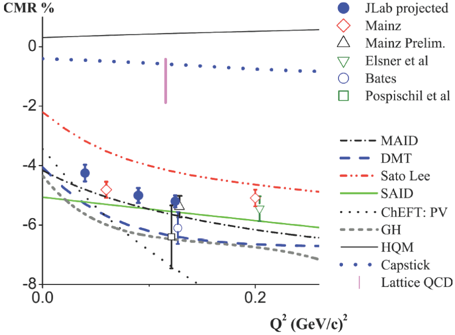




Thanks to N. Sparveris.

- I. Aznauryan *et al.*, CLAS, Phys. Rev. C 80 (2009) 055203
- New measurement of the Coulomb quadrupole amplitude in the low momentum transfer region (E08-010) , N. Sparveris *et al.*, Hall A

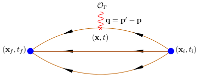
Lattice evaluation

$$\langle\Delta(p',s')|j_\mu|N(p,s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta\, m_N}{E_\Delta(\mathbf{p}')\,E_N(\mathbf{p})}\right)^{1/2}\bar{u}_\sigma(p',s')\left[G^*_{M1}(q^2)K^{M1}_{\sigma\mu}+G^*_{E2}(q^2)K^{E2}_{\sigma\mu}+G^*_{C2}K^{C2}_{\sigma\mu}\right]u(p,s)$$

- Evaluation of two-point and three-point functions

$$G(\vec{q},t) = \sum_{\vec{x}_f} e^{-i\vec{x}_f\cdot\vec{q}}\,\Gamma^4_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\bar{J}_\beta(0)\rangle$$

$$G^{\mu\nu}(\Gamma,\vec{q},t) = \sum_{\vec{x}_f,\vec{x}} e^{i\vec{x}\cdot\vec{q}}\,\Gamma_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\mathcal{O}^\mu(\vec{x},t)\bar{J}_\beta(0)\rangle$$



$$R^J_\sigma(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma_\tau;\mu) = \frac{\langle G^{\Delta J_\mu N}_\sigma(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma_\tau)\rangle}{\langle G^{\Delta\Delta}_{ii}(t_2,\mathbf{p}';\Gamma_4)\rangle}\left[\frac{\langle G^{\Delta\Delta}_{ii}(t_2,\mathbf{p}';\Gamma_4)\rangle}{\langle G^{NN}(t_2,\mathbf{p};\Gamma_4)\rangle}\,\frac{\langle G^{NN}(t_2-t_1,\mathbf{p};\Gamma_4)\rangle\,\langle G^{\Delta\Delta}_{ii}(t_1,\mathbf{p}';\Gamma_4)\rangle}{\langle G^{\Delta\Delta}_{ii}(t_2-t_1,\mathbf{p}';\Gamma_4)\rangle\,\langle G^{NN}(t_1,\mathbf{p};\Gamma_4)\rangle}\right]^{1/2}$$

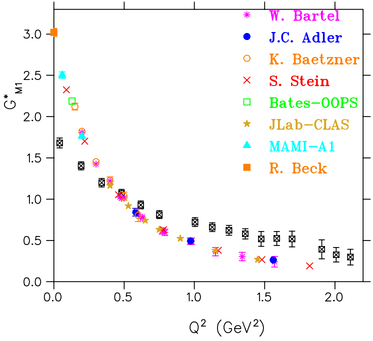
- Construct optimized sources to isolate quadrupoles $\rightarrow$ three-sequential inversions needed

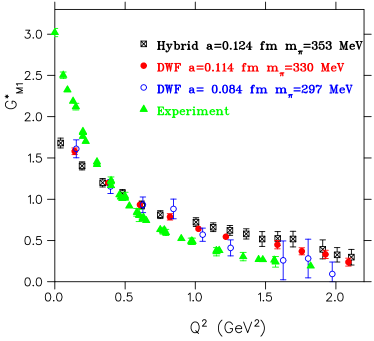
$$S^J_1(\mathbf{q};J) = \sum_{\sigma=1}^{3}\Pi^J_\sigma(\mathbf{0},-\mathbf{q};\Gamma_4;J)\quad,\qquad S^J_2(\mathbf{q};J)=\sum_{\sigma\neq k=1}^{3}\Pi^J_\sigma(\mathbf{0},-\mathbf{q};\Gamma_k;J)$$

$$S^J_3(\mathbf{q};J) = \Pi^J_3(\mathbf{0},-\mathbf{q};\Gamma_3;J)-\frac{1}{2}\left[\Pi^J_1(\mathbf{0},-\mathbf{q};\Gamma_1;J)+\Pi^J_2(\mathbf{0},-\mathbf{q};\Gamma_2;J)\right]$$

- Use the coherent sink technique: create four sets of forward propagators for each configuration at source positions separated in time by one-quarter of the total temporal size, Syritsyn et al. (LHPC), Phys. Rev. D81 (2009) 034507.

Results on magnetic dipole





Slope smaller than experiment, underestimate G_{*M1} at low Q^2 $\rightarrow$ pion cloud effects?

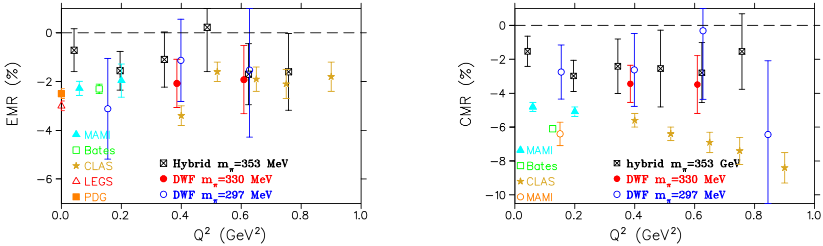
New results using $N_f = 2 + 1$ dynamical Domain Wall Fermions, simulated by RBC-UKQCD Collaborations $\Longrightarrow$ No visible improvement.
C. A., G.Koutsou, J.W. Negele, Y. Proestos, A. Tsapalis, Phys. Rev. D83 (2011)

Situation like for nucleon form factors, independent of lattice discretization

$\Longrightarrow$ nucleon FFs under study by a number of lattice groups.

Results on EMR and CMR

Systematic errors may cancel in rations: G_{E2} and G_{C2} are suppressed at low Q^2 like G^*_{M1}
$\Longrightarrow$ look at EMR and CMR



New results using $N_f = 2 + 1$ dynamical domain wall fermions by RBC-UKQCD Collaborations
Need large statistics to reduce the errors $\Longrightarrow$ as $m_\pi \to 140$ MeV $\mathcal{O}(10^3)$ need to be analyzed.

N - Δ axial-vector form factors

$$\langle \Delta(p', s')|A^3_\mu|N(p, s)\rangle = \mathcal{A}\bar{u}^\lambda(p', s')\left[\left(\frac{C^A_3(q^2)}{m_N}\gamma^\nu + \frac{C^A_4(q^2)}{m_N^2}p'^\nu\right)\left(g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu}\right)q^\rho + C^A_5(q^2)g_{\lambda\mu} + \frac{C^A_6(q^2)}{m_N^2}q_\lambda q_\mu\right]u(p, s)$$

$$\mathcal{A} = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta m_N}{E_\Delta(\mathbf{p}')E_N(\mathbf{p})}\right)^{1/2}$$

- $C^A_5(q^2)$ analogous to the nucleon $G_A(q^2)$
- $C^A_6(q^2)$, analogous to the nucleon $G_p(q^2)$ $\longrightarrow$ pion pole behaviour
- $C^A_3(q^2)$ and $C^A_4(q^2)$ are suppressed (transverse part of the axial-vector)

- Study also the pseudo-scalar transition form factor $G_{\pi N\Delta}(q^2)$
$\Longrightarrow$ Non-diagonal Goldberger-Treiman relation:

$$C^A_5(q^2) + \frac{q^2}{m_N^2}C^A_6(q^2) = \frac{1}{2m_N}\frac{G_{\pi N\Delta}(q^2)f_\pi m_\pi^2}{m_\pi^2 - q^2} \quad .$$

Pion pole dominance relates C^A_6 to $G_{\pi N\Delta}$ through:

$$\frac{1}{m_N}C^A_6(q^2) \sim \frac{1}{2}\frac{G_{\pi N\Delta}(q^2)f_\pi}{m_\pi^2 - q^2}$$

Goldberger-Treiman relation becomes

$$G_{\pi N\Delta}(q^2)\, f_\pi = 2m_N C^A_5(q^2)$$

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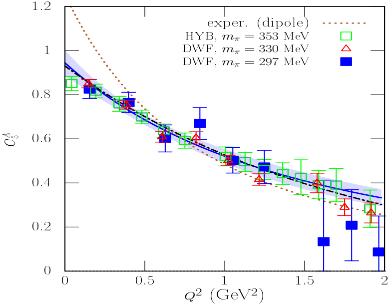
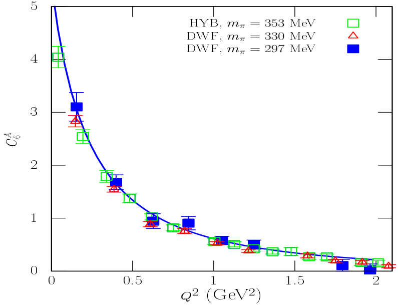
Results on Δ to N axial-vector form factors





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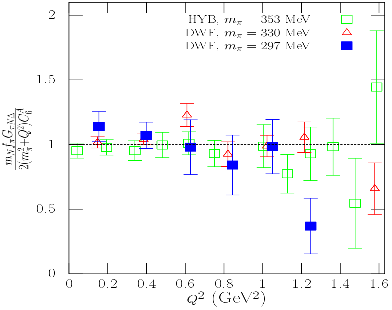
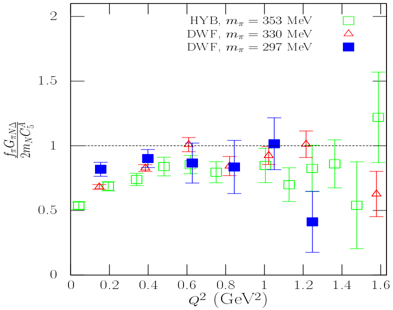




Pion-pole dominance: $\frac{1}{m_N} C_6^A(Q^2) \sim \frac{1}{2} \frac{G_{\pi N\Delta}(Q^2) f_\pi}{m_\pi^2 + Q^2}$

Goldberger-Treiman rel.: $G_{\pi N\Delta}(Q^2) f_\pi = 2 m_N C_5^A(Q^2)$

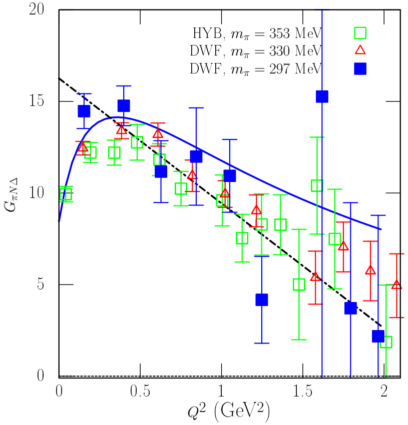
$\pi N\Delta$ pseudoscalar coupling

Pseudoscalar current: $P^a(x) = \overline{\psi}(x)\gamma_5 \frac{\tau^a}{2}\psi(x)$

- Δ-N matrix element:

$$2m_q\langle\Delta(p',s')|P^3|N(p,s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta m_N}{E_\Delta(\mathbf{p}')E_N(\mathbf{p})}\right)^{1/2}\frac{f_\pi m_\pi^2\, G_{\pi N\Delta}(Q^2)}{m_\pi^2+Q^2}\bar{u}^\nu(p',s')\frac{q_\nu}{2m_N}u(p,s)$$

- Fit to: $G_{\pi N\Delta}(Q^2) = K\,\frac{(Q^2/m_\pi^2+1)}{(Q^2/m_{C_5}^2+1)^2(Q^2/m_{C_6}^2+1)}$



In a previous study: $N_f = 2$ Wilson results consistent with $G_{\pi N\Delta}(Q^2) \sim 1.6 G_{\pi NN}(Q^2)$

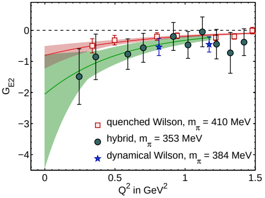
Δ electromagnetic form factors

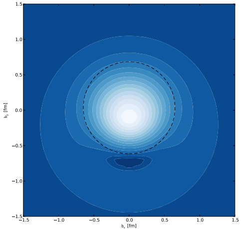
$$\langle \Delta(p',s')|j^\mu(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\left\{\left[F_1^*(Q^2)g^{\alpha\beta}+F_3^*(Q^2)\frac{q^\alpha q^\beta}{(2M_\Delta)^2}\right]\gamma^\mu+\left[F_2^*(Q^2)g^{\alpha\beta}+F_4^*(Q^2)\frac{q^\alpha q^\beta}{(2M_\Delta)^2}\right]\frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}\right\}u_\beta(p,s)$$

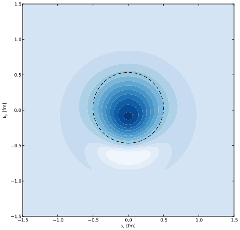
with e.g. the quadrupole form factor given by: $G_{E2}=\left(F_1^*-\tau F_2^*\right)-\frac{1}{2}(1+\tau)\left(F_3^*-\tau F_4^*\right)$, where $\tau\equiv Q^2/(4M_\Delta^2)$

Construct an optimized source to isolate G_{E2} $\rightarrow$ additional sequential propagators needed.
Neglect disconnected contributions in this evaluation.

Transverse charge density of a Δ polarized along the x-axis can be defined in the infinite momentum frame $\rightarrow$ $\rho^\Delta_{T\,\frac{3}{2}}(\vec{b})$ and $\rho^\Delta_{T\,\frac{1}{2}}(\vec{b})$.
Using G_{E2} we can predict 'shape' of Δ.







Δ with spin 3/2 projection elongated along spin axis compared to the Ω^-

C. A., T. Korzec, G. Koutsou, C. Lorcé, J. W. Negele, V. Pascalutsa, A. Tsapalis, M. Vanderhaeghen, NPA825 ,115 (2009).

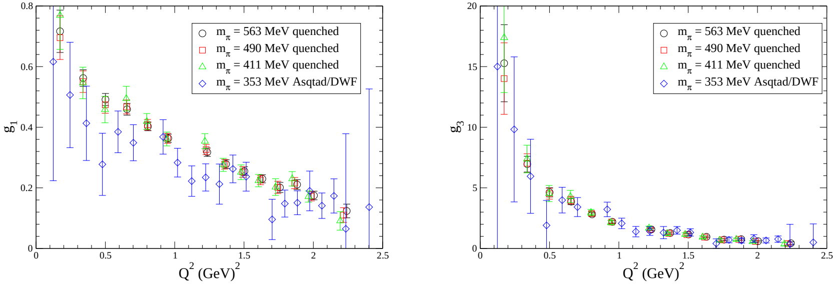
Δ axial-vector form factors

Axial-vector current: $A^a_\mu(x) = \overline{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$

$$\langle\Delta(p',s')|A^3_\mu(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\frac{1}{2}\left[-g^{\alpha\beta}\left(g_1(q^2)\gamma^\mu\gamma^5 + g_3(q^2)\frac{q^\mu}{2M_\Delta}\gamma^5\right) + \frac{q^\alpha q^\beta}{4M^2_\Delta}\left(h_1(q^2)\gamma^\mu\gamma^5 + h_3(q^2)\frac{q^\mu}{2M_\Delta}\gamma^5\right)\right]u_\beta(p,s)$$

i.e. 4 axial form-factors, g_1, g_3, h_1 and h_3 $\longrightarrow$ at $q^2 = 0$ we can extract the Δ axial charge

PRELIMINARY



⟹ Using a consistent chiral perturbation theory framework extract the chiral Lagrangian couplings g_A, c_A, g_Δ from a combined chiral fit to the lattice results on the nucleon and Δ axial charge and the axial N-to-Δ form factor $C_5(0)$.

C. A., E. Gregory, T. Korzec, G. Koutsou, J. W. Negele, T. Sato, A. Tsapalis, arXiv:1011.0411

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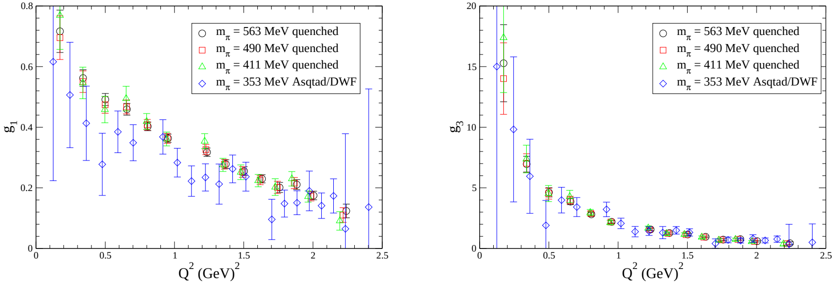

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Δ pseudoscalar couplings

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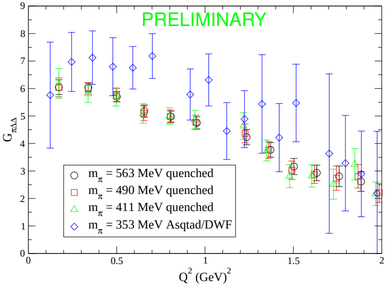
- $\Delta - \Delta$ matrix element:

$$\langle \Delta(p',s')|P^3(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\frac{1}{2}\left[-g^{\alpha\beta}\tilde{g}(q^2)\gamma^5 + \frac{q^\alpha q^\beta}{4M_\Delta^2}\tilde{h}(q^2)\gamma^5\right]u_\beta(p,s)$$

i.e. two $\pi\Delta\Delta$ couplings $\Longrightarrow$ two Goldberger-Treiman relations.

- $G_{\pi\Delta\Delta}$ is given by: $m_q\tilde{g}(Q^2) \equiv \frac{f_\pi m_\pi^2 G_{\pi\Delta\Delta}(Q^2)}{(m_\pi^2+Q^2)}$ and $H_{\pi\Delta\Delta}$ is given by: $m_q\tilde{h}(Q^2) \equiv \frac{f_\pi m_\pi^2 H_{\pi\Delta\Delta}(Q^2)}{(m_\pi^2+Q^2)}$
- Goldberger-Treiman relations: $f_\pi G_{\pi\Delta\Delta}(Q^2) = m_\Delta g_1(Q^2)$, $\quad f_\pi H_{\pi\Delta\Delta}(Q^2) = m_\Delta h_1(Q^2)$



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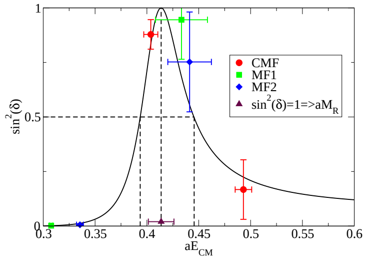
ρ-meson width

- Consider $\pi^+\pi^-$ in the $I=1$-channel
- Estimate P-wave scattering phase shift $\delta_{11}(k)$ using finite size methods
- Use Lüscher's relation between energy in a finite box and the phase in infinite volume
- Use Center of Mass frame and Moving frame
- Use effective range formula: $tan\delta_{11}(k) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{E\left(m_R^2-E^2\right)}$, $k=\sqrt{E^2/4-m_\pi^2}$ → determine M_R and $g_{\rho\pi\pi}$ and then extract $\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k_R^3}{m_R^2}$, $k_R=\sqrt{m_R^2/4-m_\pi^2}$

$m_\pi = 309$ MeV, $L = 2.8$ fm



$N_F=2$ twisted mass fermions, Xu Feng, K. Jansen, D. Renner, arXiv:0910:4891; arXiv:1011.5288

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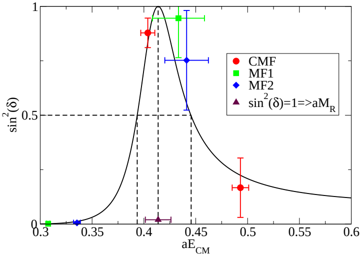

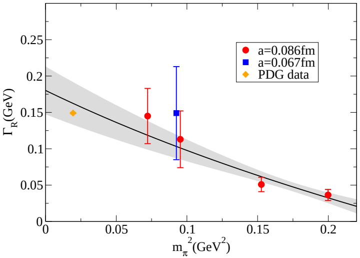

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  $\Longrightarrow$ we now have simulations of the full theory at near physical parameters
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- Nucleon form factors are being computed by a number of collaborations in order to understand the discrepancies
- N to Δ transition form factors can be extracted in a similar way to the nucleon
  Ratios of form factors expected to be less affected by lattice artifacts $\rightarrow$ EMR and CMR allow comparison to experiment
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Thank you for your attention

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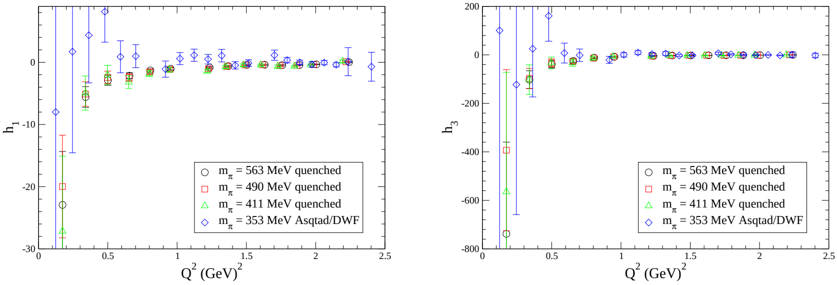

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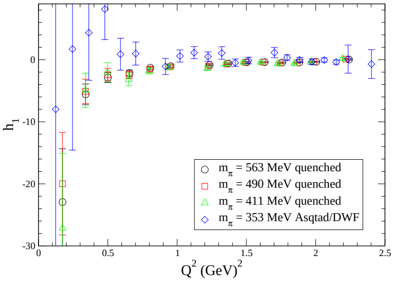

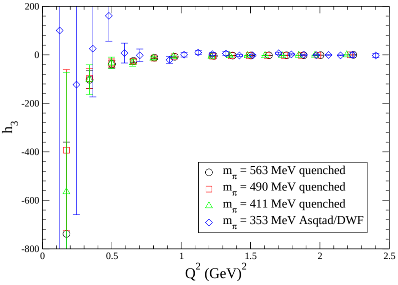

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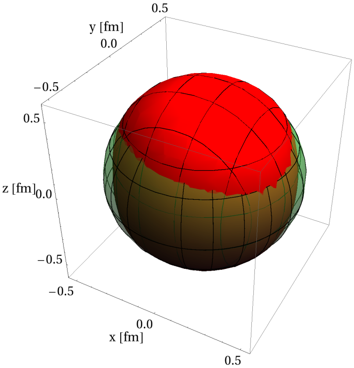
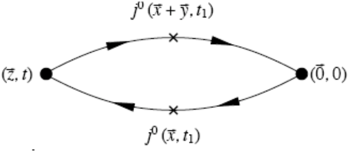
Shape of ρ-meson

Four-point functions $\stackrel{\text{non-relativistic limit}}{\longrightarrow}$ $|\psi|^2$

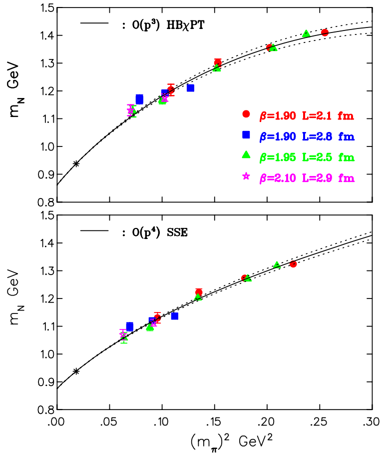




The ρ-meson having spin=1 is cigar-like in the lab frame, C. A. and G. Koutsou, Phys. Rev. D78 (2008) 094506
In agreement with form factor calculation J.N. Hedditch *et al.* Phys. Rev. D75, 094504 (2007)

Lattice spacing determination

- Use nucleon mass at physical limit
- Extrapolate using LO expansion: $m_N = m_N^0 - 4c_1 m_\pi^2 - \frac{3g_A^2}{16\pi f_\pi^2} m_\pi^3$
- Systematic error from $\mathcal{O}(p^4)$ SSE HBχPT
- Simultaneous fits to $\beta = 1.9$, $\beta = 1.95$ and $\beta = 2.1$ results



$\beta = 1.90 \quad : a = 0.087(1)(6)$
$\beta = 1.95 \quad : a = 0.078(1)(5)$
$\beta = 2.10 \quad : a = 0.060(0)(3)$